Table of Integrals, Series, and Products

Fifth Edition

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1. Introduction

1.1 Notation and Conventions used in this Table

1.1.1 Integrals and Sums

Integrals of the form $\int_a^b f(x)\,dx$ are written as $\int_a^b f(x)\,dx$ and sums of the form $\sum_{n=a}^{b} f(n)$ are written as $\sum_{n=a}^{b} f(n)$.

1.1.2 Elliptic Integrals

Elliptic integrals are written in the form given in Gradshteyn and Ryzhik, Table of Integrals, Series, and Products, Fifth Edition, Academic Press, 1994. For example, the elliptic integral of the first kind is written as $\int_0^{\pi/2} \frac{1}{\sqrt{1-k^2\sin^2 x}}\,dx$ and the elliptic integral of the second kind is written as $\int_0^{\pi/2} \sqrt{1-k^2\sin^2 x}\,dx$.

1.1.3 Bessel Functions

Bessel functions are written in the form given in Gradshteyn and Ryzhik, Table of Integrals, Series, and Products, Fifth Edition, Academic Press, 1994. For example, the Bessel function of the first kind is written as $J_\nu(z)$ and the Bessel function of the second kind is written as $Y_\nu(z)$.

1.1.4 Gamma Function

The gamma function is written as $\Gamma(z)$ and the incomplete gamma function is written as $\gamma(a,z)$.

1.1.5 Binomial Coefficients

Binomial coefficients are written as $\binom{n}{k}$.

1.1.6 Factorial Function

The factorial function is written as $n!$.

1.1.7 Pochhammer Symbol

The Pochhammer symbol is written as $(a)_n$.

1.1.8 Hypergeometric Functions

Hypergeometric functions are written as ${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$.

1.1.9 Legendre Polynomials

Legendre polynomials are written as $P_n(x)$.

1.1.10 Spherical Harmonics

Spherical harmonics are written as $Y_n^m(\theta,\phi)$.

1.1.11 Zeta Function

The Riemann zeta function is written as $\zeta(z)$.

1.1.12 Euler Constant

Euler's constant is written as γ.

1.1.13 Bernoulli Numbers

Bernoulli numbers are written as B_n.

1.1.14 Euler Numbers

Euler numbers are written as E_n.

1.1.15 Stirling Numbers

Stirling numbers of the first kind are written as $s(n,k)$ and Stirling numbers of the second kind are written as $S(n,k)$.

1.1.16 Partition Function

The partition function is written as $p(n)$.

1.1.17 Fibonacci Numbers

Fibonacci numbers are written as F_n.

1.1.18 Lucas Numbers

Lucas numbers are written as L_n.

1.1.19 Catalan Numbers

Catalan numbers are written as C_n.

1.1.20 Euler Polynomials

Euler polynomials are written as $E_n(x)$.

1.1.21 Bernoulli Polynomials

Bernoulli polynomials are written as $B_n(x)$.

1.1.22 Chebyshev Polynomials

Chebyshev polynomials are written as $T_n(x)$ and $U_n(x)$.

1.1.23 Hermite Polynomials

Hermite polynomials are written as $H_n(x)$.

1.1.24 Laguerre Polynomials

Laguerre polynomials are written as $L_n(x)$.

1.1.25 Jacobi Polynomials

Jacobi polynomials are written as $P_n^{(\alpha,\beta)}(x)$.

1.1.26 Gegenbauer Polynomials

Gegenbauer polynomials are written as $C_n^{(\lambda)}(x)$.

1.1.27 Zernike Polynomials

Zernike polynomials are written as $Z_n^m(\rho,\theta)$.

1.1.28 Spherical Bessel Functions

Spherical Bessel functions are written as $j_n(z)$ and $y_n(z)$.

1.1.29 Spherical Harmonics

Spherical harmonics are written as $Y_n^m(\theta,\phi)$.

1.1.30 Wigner 3-j Symbols

Wigner 3-j symbols are written as $\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$.

1.1.31 Wigner 6-j Symbols

Wigner 6-j symbols are written as $\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$.

1.1.32 Wigner 9-j Symbols

Wigner 9-j symbols are written as $\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & j_9 \end{Bmatrix}$.

1.1.33 Clebsch-Gordan Coefficients

Clebsch-Gordan coefficients are written as $C(j_1,j_2;m_1,m_2|j_3,m_3)$.

1.1.34 Racah Coefficients

Racah coefficients are written as $W(j_1,j_2,j_3,j_4;j_5,j_6)$.

1.1.35 Wigner D-matrix

Wigner D-matrix is written as $D_{m'm}^{j}(\alpha,\beta,\gamma)$.

1.1.36 Spherical Harmonics

Spherical harmonics are written as $Y_n^m(\theta,\phi)$.

1.1.37 Vector Spherical Harmonics

Vector spherical harmonics are written as $\mathbf{Y}_{jl}^{m}(\theta,\phi)$.

1.1.38 Tensor Spherical Harmonics

Tensor spherical harmonics are written as $\mathbf{Y}_{jl}^{m}(\theta,\phi)$.

1.1.39 Spherical Harmonics of Tensor Rank

Spherical harmonics of tensor rank are written as $\mathbf{Y}_{jl}^{m}(\theta,\phi)$.

1.1.40 Spherical Harmonics of Tensor Rank and Vector Spherical Harmonics

Spherical harmonics of tensor rank and vector spherical harmonics are written as $\mathbf{Y}_{jl}^{m}(\theta,\phi)$.

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1.2 Miscellaneous

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1.3 Formulas of Gradshteyn and Ryzhik and Other Sources

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1.3.2 Other Sources

Other sources are written as Other Sources.

2. Integrals

2.1 Integrals of Rational Functions

2.1.1 Integrals of $\frac{1}{x}$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

2.1.2 Integrals of $\frac{1}{x^2}$

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$$

2.1.3 Integrals of $\frac{1}{x^3}$

$$\int \frac{1}{x^3}\,dx = -\frac{1}{2x^2} + C$$

2.1.4 Integrals of $\frac{1}{x^4}$

$$\int \frac{1}{x^4}\,dx = -\frac{1}{3x^3} + C$$

2.1.5 Integrals of $\frac{1}{x^5}$

$$\int \frac{1}{x^5}\,dx = -\frac{1}{4x^4} + C$$

2.1.6 Integrals of $\frac{1}{x^6}$

$$\int \frac{1}{x^6}\,dx = -\frac{1}{5x^5} + C$$

2.1.7 Integrals of $\frac{1}{x^7}$

$$\int \frac{1}{x^7}\,dx = -\frac{1}{6x^6} + C$$

2.1.8 Integrals of $\frac{1}{x^8}$

$$\int \frac{1}{x^8}\,dx = -\frac{1}{7x^7} + C$$

2.1.9 Integrals of $\frac{1}{x^9}$

$$\int \frac{1}{x^9}\,dx = -\frac{1}{8x^8} + C$$

2.1.10 Integrals of $\frac{1}{x^{10}}$

$$\int \frac{1}{x^{10}}\,dx = -\frac{1}{9x^9} + C$$

2.1.11 Integrals of $\frac{1}{x^{11}}$

$$\int \frac{1}{x^{11}}\,dx = -\frac{1}{10x^{10}} + C$$

2.1.12 Integrals of $\frac{1}{x^{12}}$

$$\int \frac{1}{x^{12}}\,dx = -\frac{1}{11x^{11}} + C$$

2.1.13 Integrals of $\frac{1}{x^{13}}$

$$\int \frac{1}{x^{13}}\,dx = -\frac{1}{12x^{12}} + C$$

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